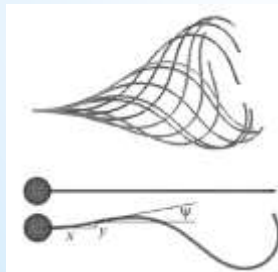
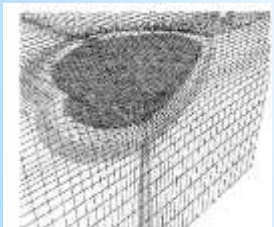


# Numerical studies of unsteady motion of continuous flows



M.A. Zaitsev



# Unsteady motion of continuous flows

Phenomena and processes under consideration:

- Nonuniform initial media condition
- Unsteady boundary condition
- Evolution of boundary
- Heat release and evolution of gas components during chemical reactions
- Buckling of solid media, instability of gas and fluid

# Application

- Advanced structural materials development
- Flameproof structures development
- Deep water pipeline design
- Definition of mechanical properties using hardness measurements data
- Heat transfer intensification structures development
- Thermal loading analysis of turbulent mixing flow in pipeline design
- Jet gas flow investigation
- Micro swimmer motion investigation

# Purpose of numerical studies

- Development and adaptation of methods, algorithms for solving relevant 2D-3D boundary problems
- Program development for different program media and databases
- Mesh generation development for high efficiency parallel computation
- Numerical solution of real practical problems

# Scientific innovativeness

- Cabaret scheme for computational modeling of linear elastic deformation problems
- Mathematical Modelling of Flagellated Microswimmers
- Cabaret method program release in OpenFOAM program media
- Cabaret method program release using CGNS database

# Numerical studies applied methods

- Cabaret method for elastic media(Cabaret)

Зайцев М.А., Карабасов С.А., Схема Кабаре для задач деформирования упругопластического тела. // Математическое моделирование РАН, 2017, том 29:11

- Integro-interpolation method(IIM)

Зайцев М.А., Гольдберг С.М. Математическое моделирование взаимодействия детонирующих сред с упругими оболочками // Математическое моделирование РАН, т. 5 N 6,1993 г.,с.56-68.

Бакиров М.Б., Зайцев М.А., Фролов И.В., Математическое моделирование процессов идентификации сферы в упругопластическое полупространство, Заводская лаборатория, 2001,67(1), 37-47.

- Finite difference method(FDM)

В.А. Петушков, М.А. Зайцев Реакция упругого тела на высокоскоростное ударное нагружение капельной средой, Машиноведение,1989,42-48

- Finite element method(FEM)

S. A. Karabasov M. A. Zaitsev, Mathematical Modelling of Flagellated Microswimmers, Computational Mathematics and Mathematical Physics, 2018,58,11,1804-1816

# Mathematical modelling of unsteady problems using the CABARET(case for elastic media)

## momentum equation

$$\rho \frac{\partial u}{\partial t} = \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z}$$

$$\rho \frac{\partial v}{\partial t} = \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z}$$

$$\rho \frac{\partial w}{\partial t} = \frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z}$$

$\rho$  - density;  $u, v, w$  velocity components;  $x, y, z$  coordinates;  $\sigma_{ij}$ -components of Cauchy stress tensor.

OpenFOAM formulation for time step  $dt2$ :

```
u=u-dt2*fvc::surfaceIntegrate(ss & mesh.Sf())/Rofon;
```

ss - stress tensor defined at faces(surfaceTensorField ss);

Rofon - density

mesh.Sf() - face vector

# Equation of state

$$\frac{\partial \sigma_{xx}}{\partial t} = 2\mu \frac{\partial u}{\partial x} + \lambda \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right)$$

$$\frac{\partial \sigma_{yy}}{\partial t} = 2\mu \frac{\partial v}{\partial y} + \lambda \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right)$$

$$\frac{\partial \sigma_{zz}}{\partial t} = 2\mu \frac{\partial w}{\partial z} + \lambda \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right)$$

$$\frac{\partial \sigma_{xy}}{\partial t} = \mu \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)$$

$$\frac{\partial \sigma_{xz}}{\partial t} = \mu \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right)$$

$$\frac{\partial \sigma_{yz}}{\partial t} = \mu \left( \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right)$$

$\lambda, \mu$  - Lamé constants of elastic media;

OpenFOAM formulation for time step dt2:

```
volTensorField gradU = fvc::surfaceIntegrate(us*mesh.Sf());  
s=s-dt2*(2.0*mu*symm(gradU)+lambda*I*tr(gradU));
```

s - stress tensor in cells;

us - velocity vector in faces(surfaceVectorField us);

lambda, mu - Lamé constants



# Eigen values

Plane problem in X direction

$$\frac{\partial}{\partial t} \begin{Bmatrix} u \\ v \\ \sigma_{xx} \\ \sigma_{xy} \\ \sigma_{yy} \end{Bmatrix} + \begin{Bmatrix} 0 & 0 & -\frac{1}{\rho} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{\rho} & 0 \\ -\lambda - 2\mu & 0 & 0 & 0 & 0 \\ 0 & -\mu & 0 & 0 & 0 \\ -\lambda & 0 & 0 & 0 & 0 \end{Bmatrix} \frac{\partial}{\partial x} \begin{Bmatrix} u \\ v \\ \sigma_{xx} \\ \sigma_{xy} \\ \sigma_{yy} \end{Bmatrix} + \begin{Bmatrix} 0 & 0 & 0 & -\frac{1}{\rho} & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{\rho} \\ 0 & -\lambda & 0 & 0 & 0 \\ -\mu & 0 & 0 & 0 & 0 \\ 0 & -\lambda - 2\mu & 0 & 0 & 0 \end{Bmatrix} \frac{\partial}{\partial y} \begin{Bmatrix} u \\ v \\ \sigma_{xx} \\ \sigma_{xy} \\ \sigma_{yy} \end{Bmatrix} = 0$$

$$\det \begin{Bmatrix} 0 - \Lambda & 0 & -\frac{1}{\rho} & 0 & 0 \\ 0 & 0 - \Lambda & 0 & -\frac{1}{\rho} & 0 \\ -\lambda - 2\mu & 0 & 0 - \Lambda & 0 & 0 \\ 0 & -\mu & 0 & 0 - \Lambda & 0 \\ -\lambda & 0 & 0 & 0 & 0 - \Lambda \end{Bmatrix} = 0 \quad \Lambda^5 - \Lambda^3 \frac{1}{\rho} (\lambda + 3\mu) + \Lambda \frac{1}{\rho^2} (\lambda + 2\mu) \mu = 0$$

# Invariants

$$I_+ = u - \frac{\sigma_{xx}}{\rho c_1}$$

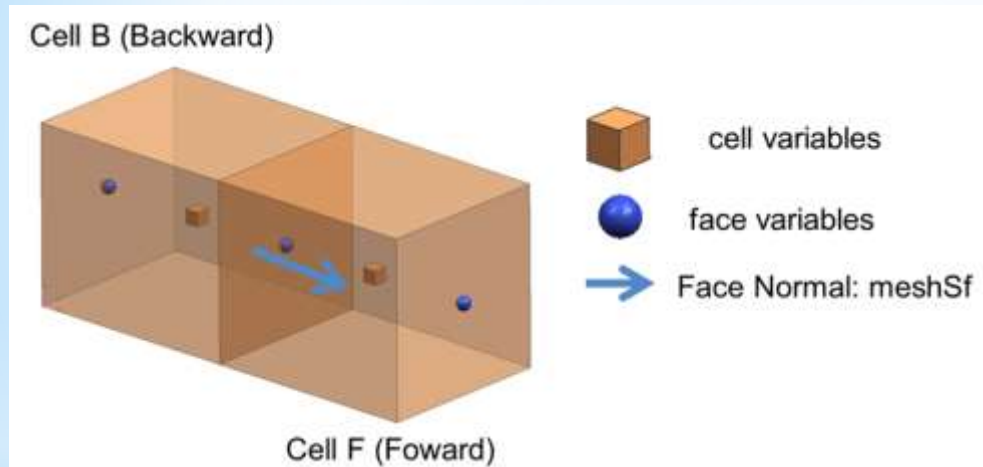
$$I_- = u + \frac{\sigma_{xx}}{\rho c_1}$$

$$J_+ = v - \frac{\sigma_{xy}}{\rho c_2}$$

$$J_- = v + \frac{\sigma_{xy}}{\rho c_2}$$

Eigen values are equal positive and negative values of longitudinal and transverse wave velocity. Zero eigen value is for Y-direction stress invariant.

# Space and time stencils



OpenFOAM formulation:

Time loop

Phase 1;

Phase 2;

Phase 3;

Loop end

Phase 2 is external function.

Boundaries are OpenFOAM

codedMixed type.

# Cell computations( Phase 1 & 3)

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix}_{new} = \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} + \frac{\Delta t}{2\rho V} \sum_{k=1}^6 \begin{Bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{Bmatrix} \begin{Bmatrix} S_x \\ S_y \\ S_z \end{Bmatrix}$$

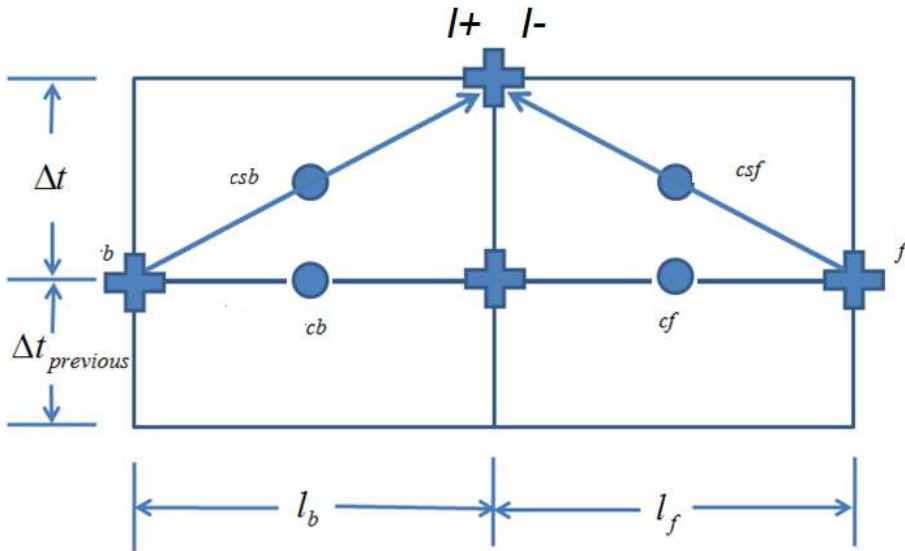
$$\begin{Bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{Bmatrix} = \frac{1}{V} \sum_{k=1}^6 \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} \{S_x \quad S_y \quad S_z\}$$

V - cell volume,  $\Delta t$  - time step.

$$\Delta = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{xy} \\ \sigma_{xz} \\ \sigma_{yy} \\ \sigma_{yz} \\ \sigma_{zz} \end{Bmatrix}_{new} = \begin{Bmatrix} \sigma_{xx} \\ \sigma_{xy} \\ \sigma_{xz} \\ \sigma_{yy} \\ \sigma_{yz} \\ \sigma_{zz} \end{Bmatrix} + \frac{\Delta t}{2} \begin{Bmatrix} \lambda\Delta + 2\mu \frac{\partial u}{\partial x} \\ \mu \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \\ \mu \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) \\ \lambda\Delta + 2\mu \frac{\partial v}{\partial y} \\ \mu \left( \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right) \\ \lambda\Delta + 2\mu \frac{\partial w}{\partial z} \end{Bmatrix}$$

# Face computations



$c$  - invariant value,  $l$  - distance to opposite face, indices  $b$  и  $f$  are for backward and forward face invariant value,  $cb$  и  $cf$  are for backward and forward cell invariant value,  $csb$  и  $csf$  are for backward and forward cell invariant value on intermediate time step.

$$I_+^{\max} = \max(I, I_b, I_{cb}) + 2(I_{csb} - I_{cb}) + c \frac{\Delta t}{l} (I - I_b)$$

$$I_+^{\min} = \min(I, I_b, I_{cb}) + 2(I_{csb} - I_{cb}) + c \frac{\Delta t}{l} (I - I_b)$$

$$I_+^{\text{new}} = \begin{cases} I_+^{\max} & 2I_{csb} - I_b > I_+^{\max} \\ 2I_{csb} - I_b & I_+^{\min} \leq 2I_{csb} - I_b \leq I_+^{\max} \\ I_+^{\min} & 2I_{csb} - I_b < I_+^{\min} \end{cases}$$

$$I_-^{\max} = \max(I, I_f, I_{cf}) + 2(I_{csf} - I_{cf}) + c \frac{\Delta t}{l} (I_f - I)$$

$$I_-^{\min} = \min(I, I_f, I_{cf}) + 2(I_{csf} - I_{cf}) + c \frac{\Delta t}{l} (I_f - I)$$

$$I_-^{\text{new}} = \begin{cases} I_-^{\max} & 2I_{csb} - I_b > I_-^{\max} \\ 2I_{csf} - I_f & I_-^{\min} \leq 2I_{csb} - I_b \leq I_-^{\max} \\ I_-^{\min} & 2I_{csb} - I_b < I_-^{\min} \end{cases}$$

New velocity values - half summ of invariants with indices “+” and “-”. Stresses - half difference, multiplied by factor  $pc$ .

# Integro-interpolation method(IIM)

$$\rho \ddot{x}_i = \sigma_{ij,j} + \rho f_i$$

$$v_k \frac{\partial \sigma_{ij}}{\partial x_k} + \frac{\partial \sigma_{ij}}{\partial t} = \dot{\sigma}_{ij} = \sigma_{ij}^{\nabla} + \sigma_{ik} \Omega_{kj} + \sigma_{jk} \Omega_{ki}$$

$$\Omega_{ij} = \frac{1}{2} \left( \frac{\partial v_j}{\partial x_i} - \frac{\partial v_i}{\partial x_j} \right)$$

$$\sigma_{ij}^{\nabla} = C_{ijkl} \dot{\epsilon}_{kl}$$

$$C_{ijkl} = \left( K - \frac{3}{2} G \right) \delta_{ij} \delta_{kl} + 2G \delta_{ik} \delta_{jl}$$

$$\dot{\epsilon}_{ij} = \frac{1}{2} \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right)$$

$$\sum_{m=1}^M \left\{ \int_V \rho (\mathbf{N}^T \mathbf{N})_a dV + \int_{V_m} \mathbf{B}^T \bar{\sigma} dV - \int_{V_m} \rho \mathbf{N}^T \mathbf{f} dV - \int_{\partial b_1} \mathbf{N}^T \mathbf{g} ds \right\}^m = 0$$

$$\mathbf{M} \mathbf{a}^n = \mathbf{P}^n - \mathbf{F}^n + \mathbf{H}^n$$

$$\mathbf{a}^n = \mathbf{M}^{-1} (\mathbf{P}^n - \mathbf{F}^n + \mathbf{H}^n)$$

$$\mathbf{v}^{n+1/2} = \mathbf{v}^{n-1/2} + \mathbf{a}^n \Delta t^n$$

$$\mathbf{x}^{n+1} = \mathbf{x}^n + \mathbf{v}^{n+1/2} \Delta t^{n+1/2}$$

$$\Delta t^{n+1/2} = (\Delta t^n + \Delta t^{n+1}) / 2$$

Дробышевский Н.И., Зайцев М.А., Филлипов А.С., Конечно-элементное теплого и механического воздействия на объекты и конструкции атомной техники. Препринт ИБРАЭ, 1993, с.1-18.

Зайцев М.А., Гольдберг С.М. Математическое моделирование взаимодействия детонирующих сред с упругими оболочками // Математическое моделирование РАН, т. 5 N 6, 1993 г., с.56-68.

Бакиров М.Б., Зайцев М.А., Фролов И.В., Математическое моделирование процессов идентифирования сферы в упругопластическое полупространство, Заводская лаборатория, 2001, 67(1), 37-47.

# Cabaret method program release in OpenFOAM program media

## Elastic media

OpenFOAM formulation for time step dt2:

```
volTensorField gradU = fvc::surfaceIntegrate(us*mesh.Sf());  
s=s-dt2*(2.0*mu*symm(gradU)+lambda*I*tr(gradU));
```

s - stress tensor in cells;

us - velocity vector in faces(surfaceVectorField us);

lambda, mu - Lamé constants

## Linear compressible fluid

```
p=p-dt2*rss*fvc::surfaceIntegrate(mesh.Sf() & us);  
u=u-dt2*fvc::surfaceIntegrate((mesh.Sf() & us)*us+ps*mesh.Sf()/Rofon)  
+dt2*fvc::laplacian(nu, u)+g*dt2*t;
```

Зайцев М.А., Карабасов С.А., Математическое моделирование нестационарных задач движения сплошной среды методом Кабаре с использованием СПО OpenFOAM, «ОТКРЫТАЯ КОНФЕРЕНЦИЯ ИСП РАН ИМ. В.П. ИВАННИКОВА», Москва, 2017 г.

# Cabaret method program release using CGNS database

- Cabaret method program release using CGNS database CGNS 3.21 for moving mesh.
- Base of CGNS 3.21 format are HDF5 utilities.
- Parallel computation initial data preprocessor core is metis algorithm
- Metis utilite output are node and cell arrays for processors.
- CGNS “Base” subdirectory for each processor has “ZAlxx “ name. Subdirectory contains mesh information, boundary conditions number including processor interface data.

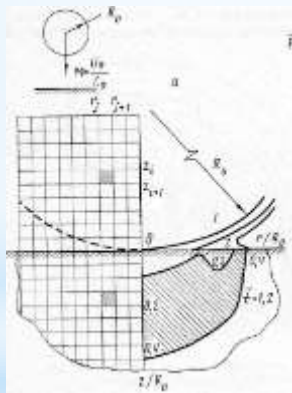


# Advanced structural materials development(FDM)

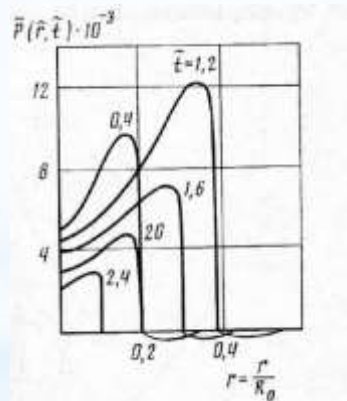
$$\begin{aligned}
 (\lambda+2G) \left( \frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r} \frac{\partial u_r}{\partial r} - \frac{u_r}{r^2} + \frac{\partial^2 u_z}{\partial r \partial z} \right) + G \left( \frac{\partial^2 u_r}{\partial z^2} - \frac{\partial^2 u_z}{\partial r \partial z} \right) &= \rho \frac{\partial^2 u_r}{\partial t^2} \\
 (\lambda+2G) \left( \frac{\partial^2 u_z}{\partial z^2} + \frac{1}{r} \frac{\partial u_r}{\partial z} + \frac{\partial^2 u_r}{\partial z \partial r} \right) + \\
 + G \left( \frac{\partial^2 u_z}{\partial r^2} - \frac{\partial^2 u_r}{\partial z \partial r} - \frac{1}{r} \frac{\partial u_r}{\partial z} + \frac{1}{r} \frac{\partial u_z}{\partial r} \right) &= \rho \frac{\partial^2 u_z}{\partial t^2}, \quad (r, z) \in D,
 \end{aligned}$$

$$\begin{aligned}
 A_{i,m}^{n+1} &= 2A_{i,m}^n - A_{i,m}^{n-1} + \left( \frac{\Delta t}{\Delta r} \right)^2 \left[ (A_{i+1,m}^n - 2A_{i,m}^n + A_{i-1,m}^n) + \right. \\
 &+ \frac{G}{\lambda+2G} (A_{i,m+1}^n - 2A_{i,m}^n + A_{i,m-1}^n) \left( \frac{\Delta r}{\Delta z} \right)^2 + \frac{1}{2l} (A_{i+1,m}^n - A_{i-1,m}^n) - \\
 &- \frac{1}{l} A_{i,m}^n + \frac{1}{4} \left( 1 - \frac{G}{\lambda+2G} \right) \left( \frac{\Delta t}{\Delta z} \right) (B_{i+1,m+1}^n - B_{i+1,m-1}^n - B_{i-1,m+1}^n + B_{i-1,m-1}^n) \left. \right], \\
 B_{i,m}^{n+1} &= 2B_{i,m}^n - B_{i,m}^{n-1} + \left( \frac{\Delta t}{\Delta r} \right)^2 \left[ (B_{i,m+1}^n - 2B_{i,m}^n + B_{i,m-1}^n) \left( \frac{\Delta r}{\Delta z} \right)^2 + \right. \\
 &+ \frac{1}{2l} \left( 1 - \frac{G}{\lambda+2G} \right) \left( \frac{\Delta r}{\Delta z} \right)^2 (A_{i,m+1}^n - A_{i,m-1}^n) + \frac{1}{4} \left( 1 - \frac{G}{\lambda+2G} \right) \left( \frac{\Delta r}{\Delta z} \right) \times \\
 &\times (A_{i+1,m+1}^n - A_{i+1,m-1}^n - A_{i-1,m+1}^n + A_{i-1,m-1}^n) + \frac{G}{\lambda+2G} (B_{i+1,m}^n - 2B_{i,m}^n + B_{i-1,m}^n) \left. \right],
 \end{aligned}$$

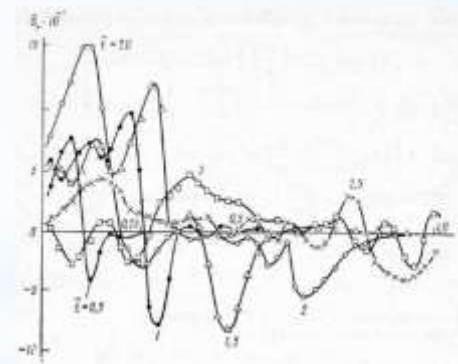
где  $A = u_r/R_0$ ,  $B = u_z/R_0$ ,  $A^n = A(n\Delta t)$ ,  $N\Delta t = \tau$ ,  $0 \leq n \leq N$ .



Geometry and boundary condition

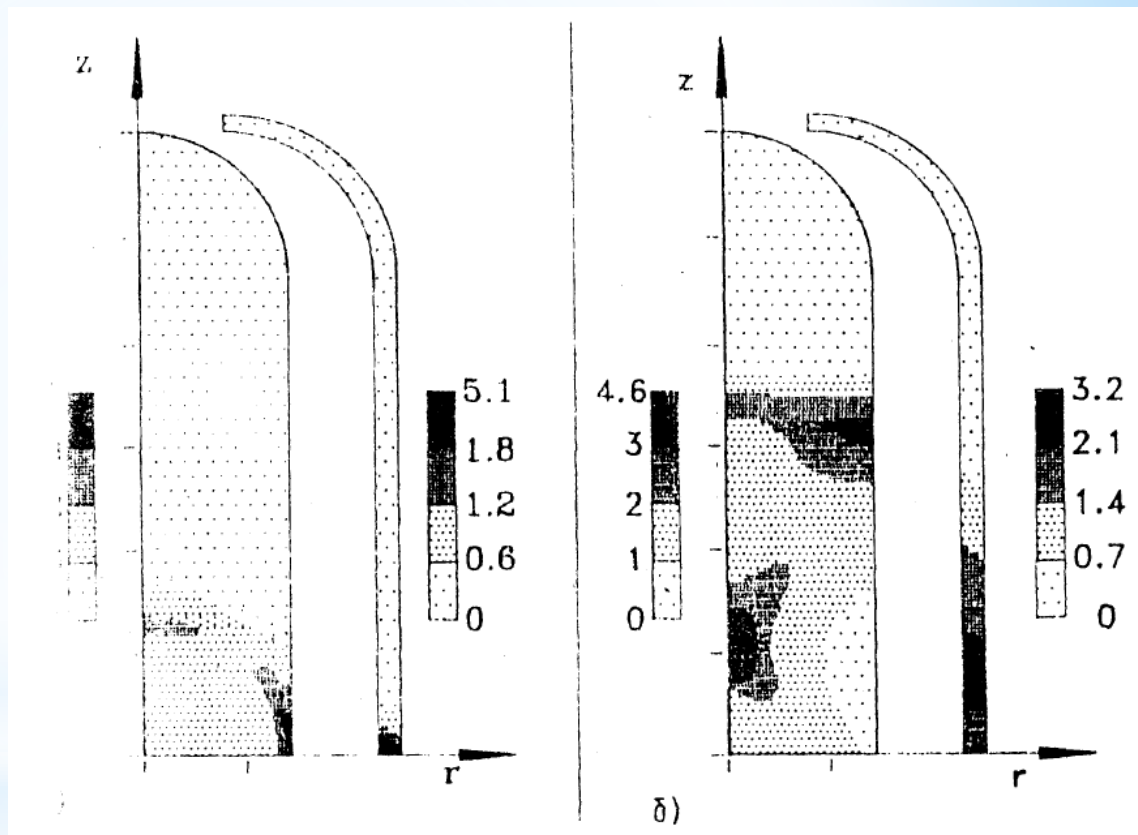
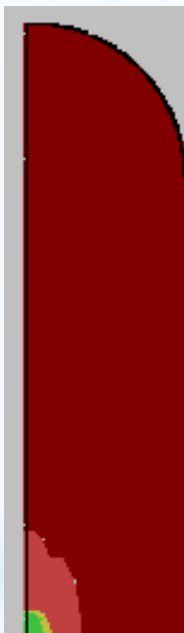
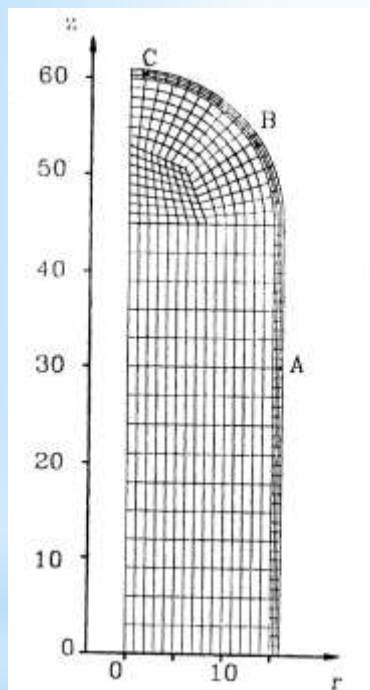


Applied external pressure



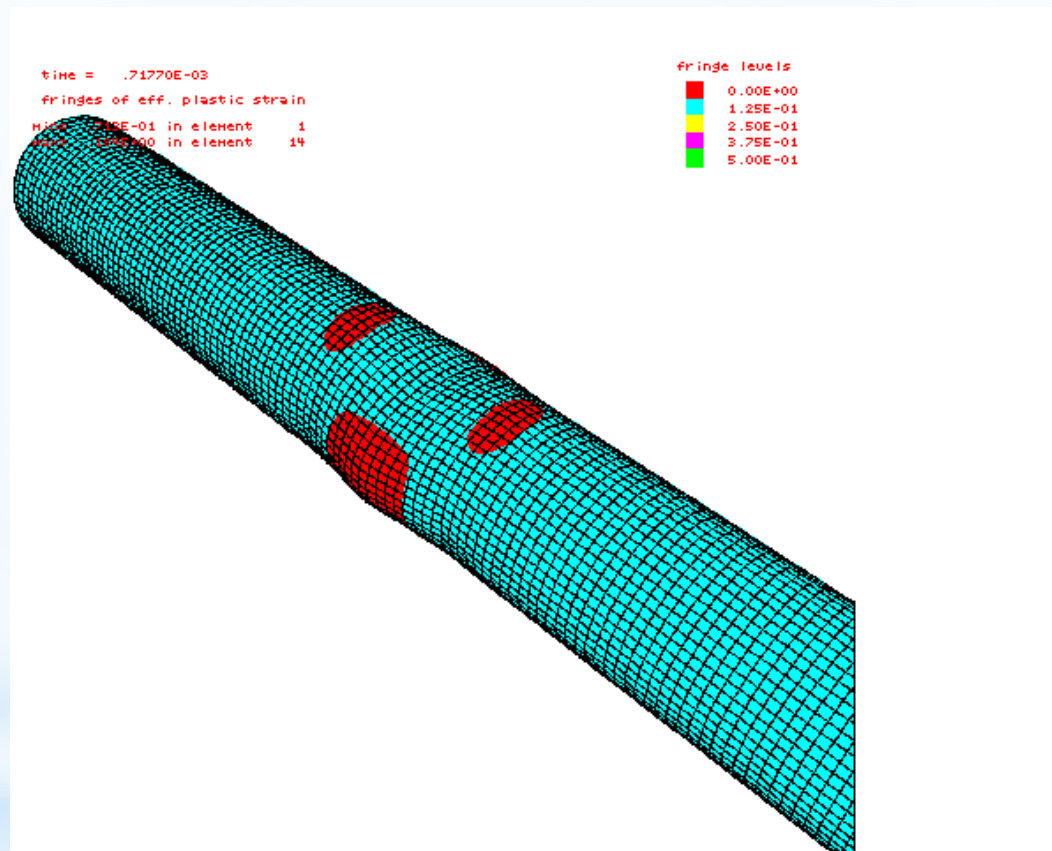
Radial stress versus time

# Flameproof structures development(IIM)



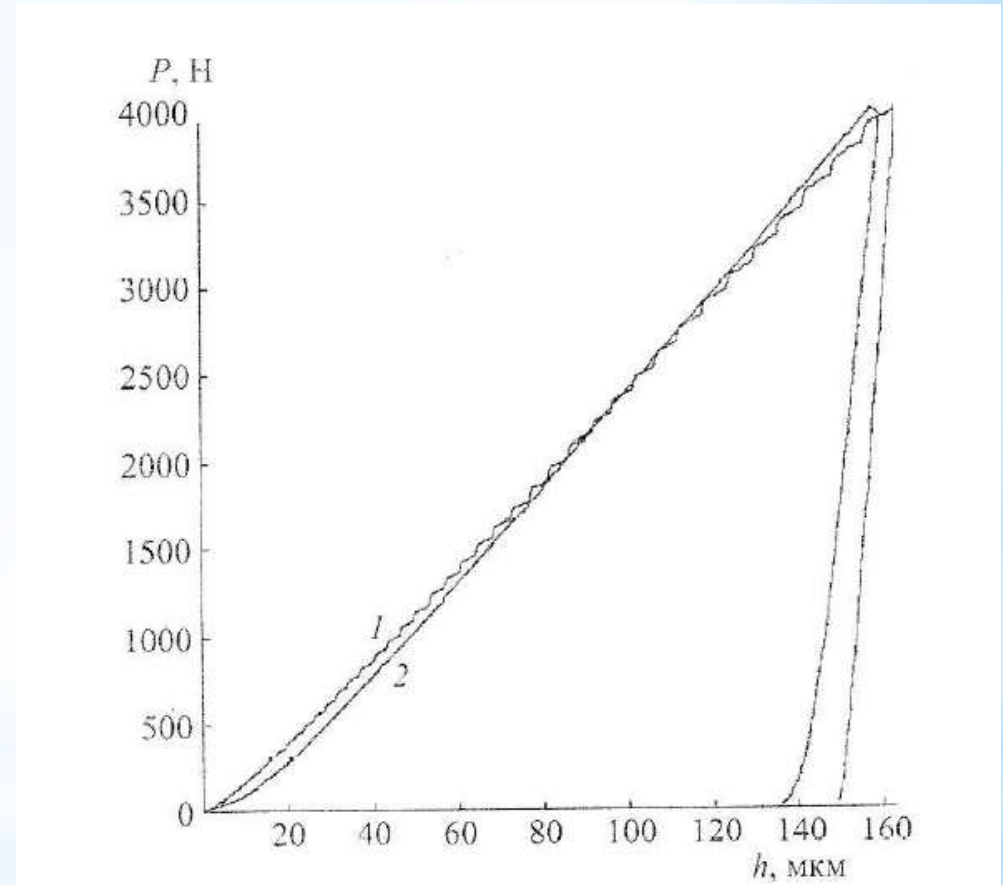
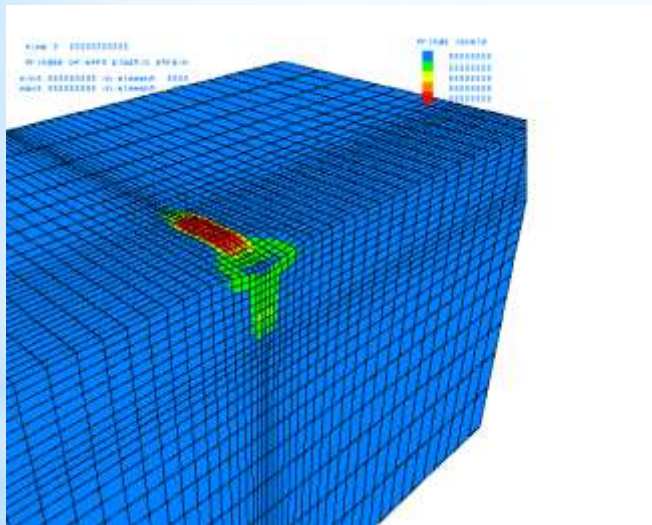
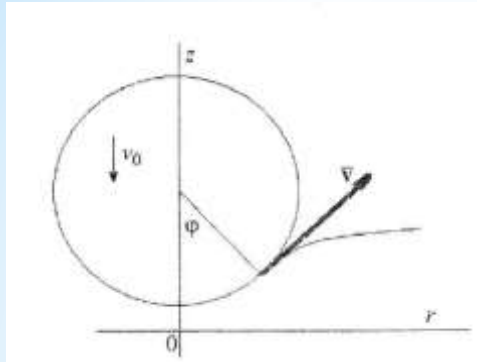
Зайцев М.А., Гольдберг С.М. Математическое моделирование взаимодействия детонирующих сред с упругими оболочками // Математическое моделирование РАН, т. 5 N 6, 1993 г., с.56-68.

# Deep water pipeline design(IIM)



Головизнин В., Зайцев М., Киселев В., Тутнов И., Математическое моделирование напряженно-деформированного состояния глубоководных морских трубопроводов. Труды третьей международной конференции по безопасности трубопроводов, Москва, 1999, с. 129-137

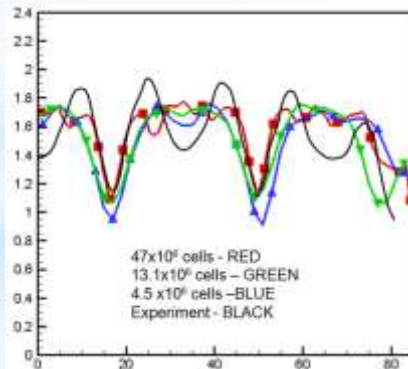
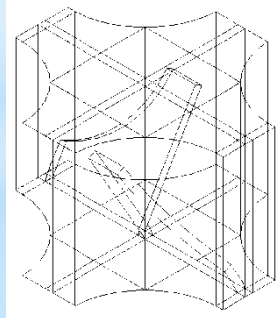
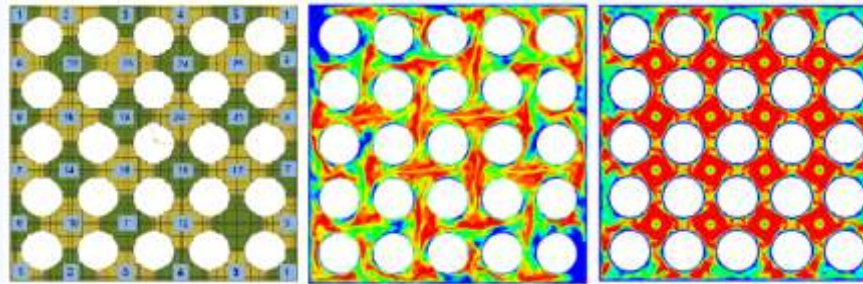
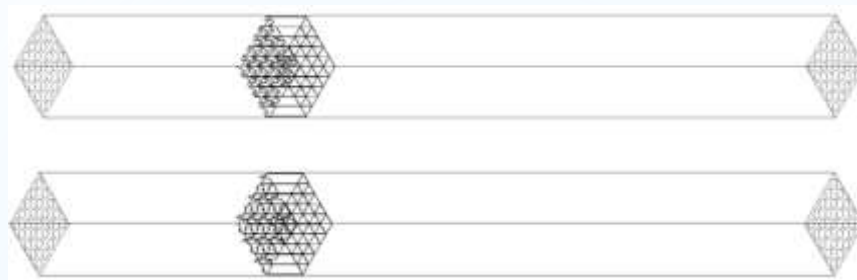
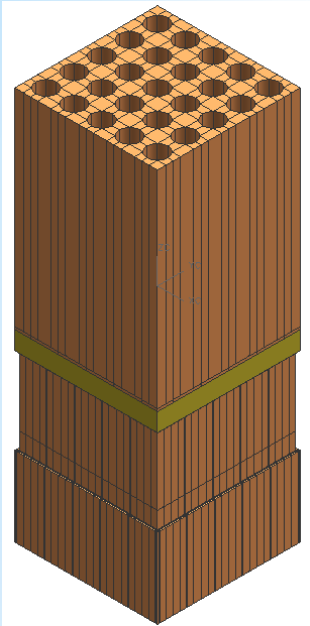
# Definition of mechanical properties using hardness measurements data (IIM, Cabaret)



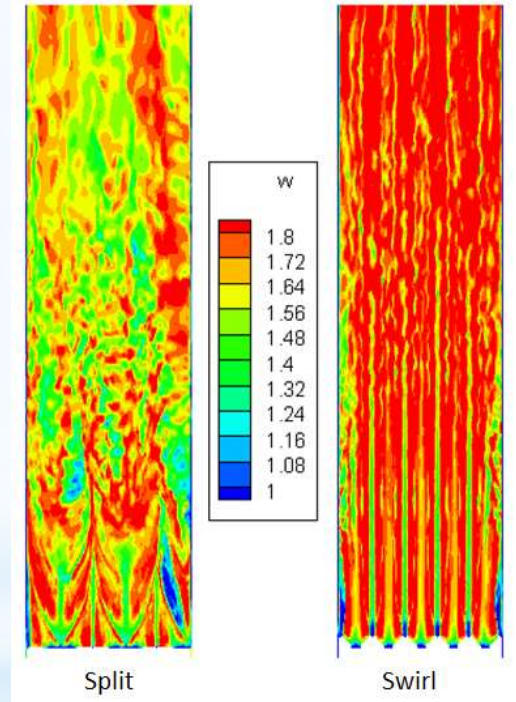
Бакиров М.Б., Зайцев М.А., Фролов И.В., Математическое моделирование процессов indentирования сферы в упругопластическое полупространство, Заводская лаборатория, 2001,67(1), 37-47.

Зайцев М.А., Карабасов С.А., Схема Кабаре для задач деформирования упругопластического тела.// Математическое моделирование РАН, 2017, том 29:11

# Heat transfer intensification structures development(Cabaret)

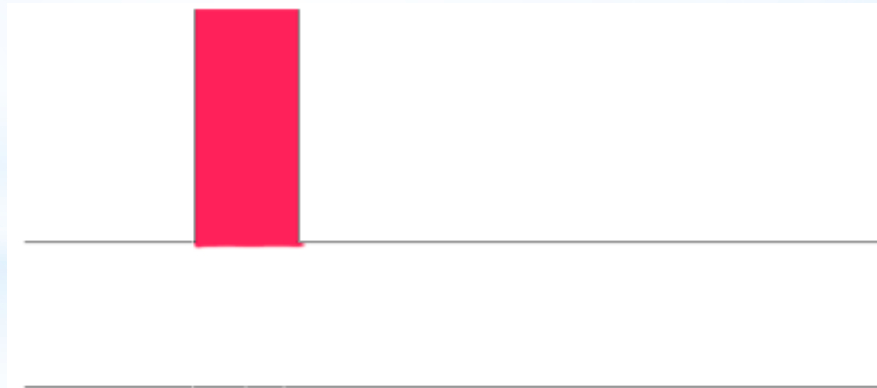
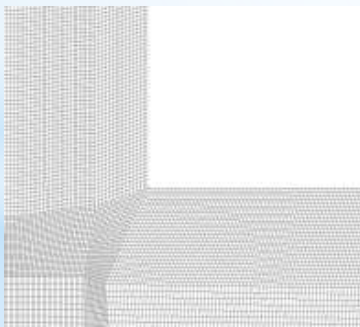
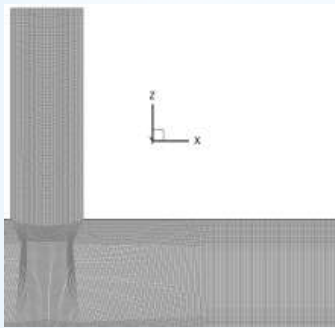


Axial velocity comparison for swirl,  $y/p=0.5$  and  $z=0.5Dh$  with different meshes (grids =4.5, 13.1, and  $47 \times 10^6$  cells).



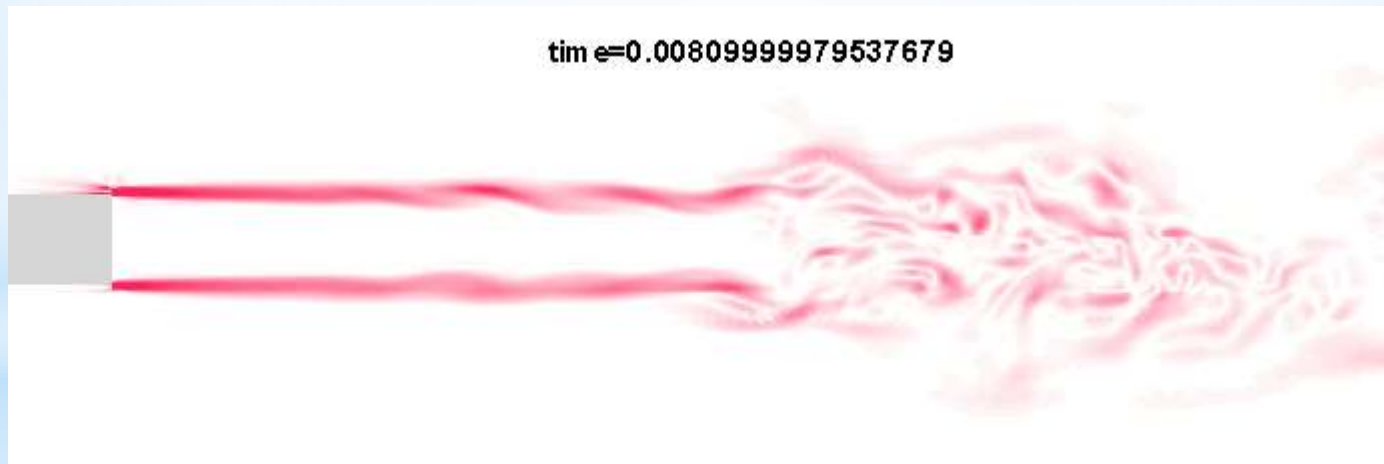
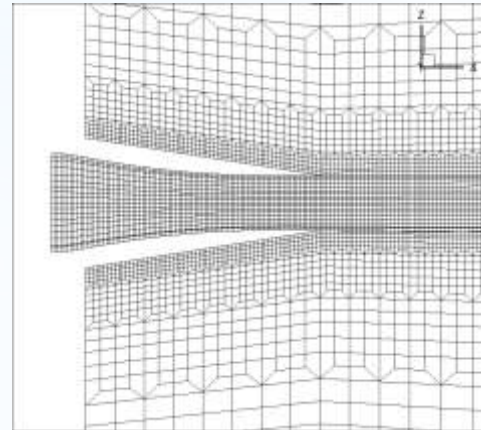
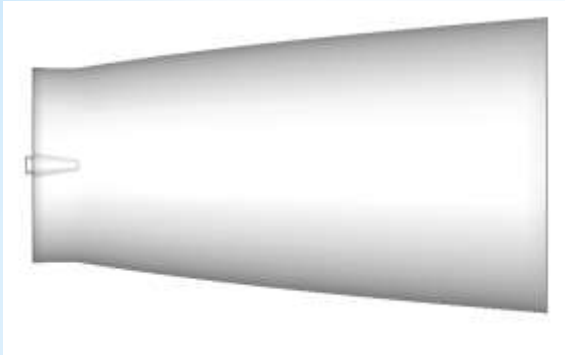
Goloviznin, V.M., Zaitsev, M.A., and Karabasov, S.A. , A HIGHLY SCALABLE HYBRID MESH CABARET MILES METHOD FOR MATIS-H PROBLEM, CFD for Nuclear Reactor Safety Applications (CFD4NRS-4), South Korea, September 2012.

# Thermal loading analysis of turbulent mixing flow in pipeline design(Cabaret)



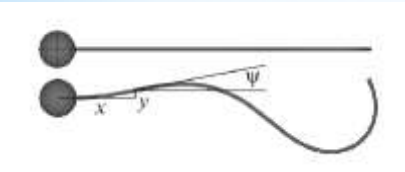
Goloviznin, V.M., Zaitsev, M.A., and Karabasov, S.A. , A HIGHLY SCALABLE HYBRID MESH CABARET MILES METHOD FOR MATIS-H PROBLEM, CFD for Nuclear Reactor Safety Applications (CFD4NRS-4), South Korea, September 2012.

# Jet gas flow investigation(Cabaret)



Faranosov G.A., Goloviznin V.M., Karabasov S.A., Zaitsev M.A., Kondakov V.G., Kopiev V.F. Cabaret method on unstructured hexahedral grids for jet noise computation // Computers and Fluids. 2013. 88. 165-179.

# Micro swimmer motion investigation(FEM)



$$\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z} = 0,$$

$$\frac{\partial p}{\partial x} = \rho_0 v \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right),$$

$$\frac{\partial p}{\partial y} = \rho_0 v \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right),$$

$$\frac{\partial p}{\partial z} = \rho_0 v \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right),$$

$$J(u, v, w) = \lambda \int_V (\Delta)^2 dV + 2\mu \int_V \left( \epsilon_{xx}^2 + \epsilon_{yy}^2 + \epsilon_{zz}^2 + \frac{1}{2} \epsilon_{xy}^2 + \frac{1}{2} \epsilon_{xz}^2 + \frac{1}{2} \epsilon_{yz}^2 \right) dV - \int_V (f_x u + f_y v + f_z w) dV.$$

$$(\Delta, \epsilon_{xx}, \epsilon_{yy}, \epsilon_{zz}, \epsilon_{xy}, \epsilon_{xz}, \epsilon_{yz}) = \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}, \frac{\partial u}{\partial x} \cdot \frac{\partial v}{\partial y}, \frac{\partial v}{\partial y} \cdot \frac{\partial w}{\partial z}, \frac{1}{2} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right), \frac{1}{2} \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right), \frac{1}{2} \left( \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right) \right).$$

$$c_x^{\dot{x}}(t)v_x^f + c_x^{\dot{y}}(t)v_y^f + c_x^{\dot{\theta}}(t)\dot{\theta}_g = -F_x(t),$$

$$c_y^{\dot{x}}(t)v_x^f + c_y^{\dot{y}}(t)v_y^f + c_y^{\dot{\theta}}(t)\dot{\theta}_g = -F_y(t),$$

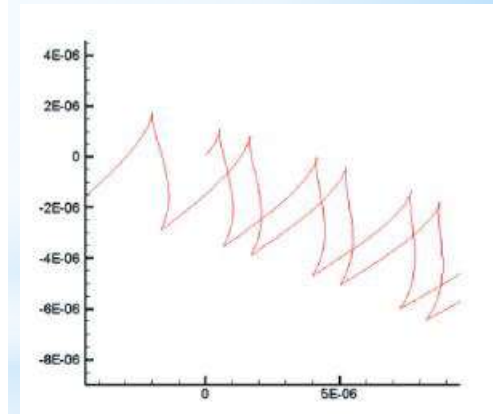
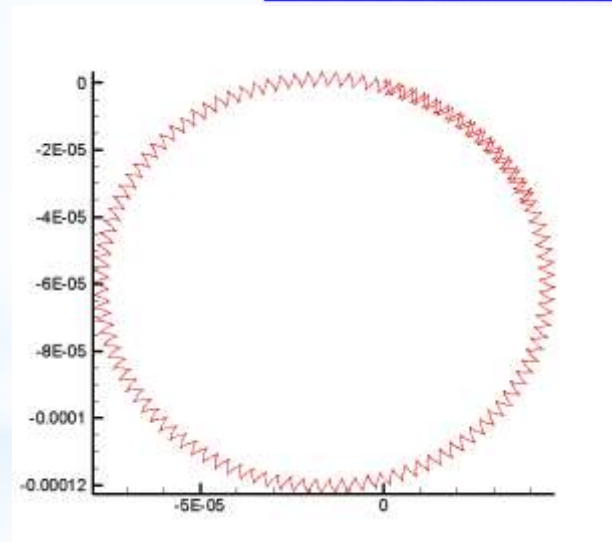
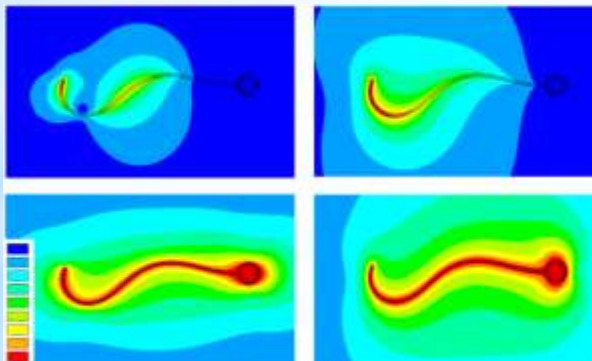
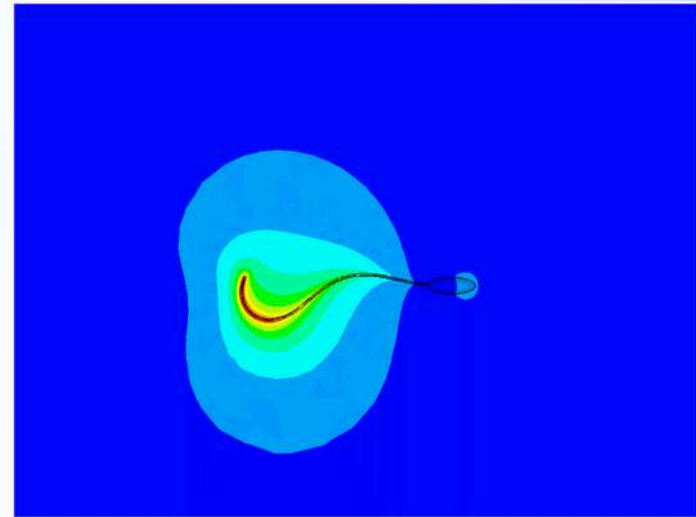
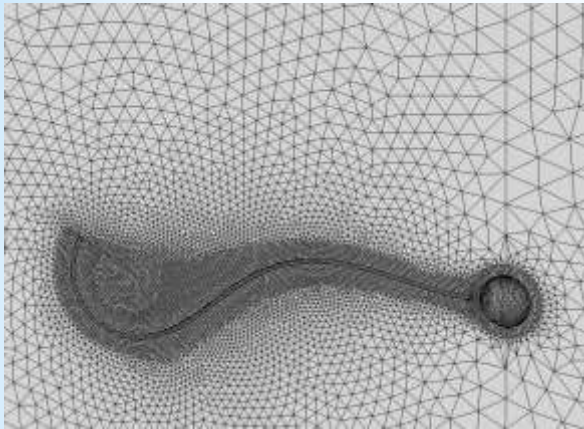
$$m_z^{\dot{x}}(t)v_x^f + m_z^{\dot{y}}(t)v_y^f + m_z^{\dot{\theta}}(t)\dot{\theta}_g = -M_z(t).$$

Motion	Force in the direction of x	Force in the direction of y	Torque about z
Shape variation	$F_x = -0.1305743 \times 10^{-7}$	$F_y = -0.1396725 \times 10^{-7}$	$M_z = -0.1284742 \times 10^{-12}$
About axis z	$c_x^{\dot{\theta}} = -0.9717138 \times 10^{-10}$	$c_y^{\dot{\theta}} = -0.1291808 \times 10^{-8}$	$m_z^{\dot{\theta}} = -0.4419866 \times 10^{-13}$
Along axis x	$c_x^{\dot{x}} = -0.5403759 \times 10^{-4}$	$c_y^{\dot{x}} = -0.1376548 \times 10^{-5}$	$m_z^{\dot{x}} = -0.9716937 \times 10^{-10}$
Along axis y	$c_x^{\dot{y}} = -0.1376548 \times 10^{-5}$	$c_y^{\dot{y}} = -0.6968351 \times 10^{-4}$	$m_z^{\dot{y}} = -0.1291808 \times 10^{-8}$

S. A. Karabasov M. A. Zaitsev, Mathematical Modelling of Flagellated Microswimmers, Computational Mathematics and Mathematical Physics, 2018,58,11,1804-1816



# Micro swimmer motion investigation results(FEM)



S. A. Karabasov M. A. Zaitsev, Mathematical Modelling of Flagellated Microswimmers, Computational Mathematics and Mathematical Physics, 2018,58,11,1804-1816  
C Rorai, M Zaitsev, S Karabasov, On the limitations of some popular numerical models of flagellated microswimmers: importance of long-range forces and flagellum waveform, Royal Society open science, 2019, 180745

# Conclusions

- Cabaret scheme for computational modeling of linear elastic deformation problems was proposed.
- Mathematical Modelling of Flagellated Microswimmers algorithm based on FEM method was proposed.
- Detailed numerical studies in areas of advanced structural materials development, flameproof structures development, deep water pipeline design, definition of mechanical properties using hardness measurements data, heat transfer intensification structures development, thermal loading analysis of turbulent mixing flow in pipeline design, jet gas flow investigation, micro swimmer motion investigation were conducted.

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**Thanks !**